

Derivatives of Inverse Trigonometric Functions

Notations for inverse trig functions:

$$\sin^{-1}(x) = \arcsin x$$

$$\tan^{-1}(x) = \arctan x$$

* does not mean that the trig function is being raised to the negative one power

$$\frac{d}{dx} \arcsin(u) = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} \arctan(u) = \frac{u'}{1+u^2}$$

Examples:

Find the derivative for each of the following.

1. $f(x) = \arcsin(x^2)$

$$f'(x) = \frac{2x}{\sqrt{1-(x^2)^2}}$$

$$f'(x) = \frac{2x}{\sqrt{1-x^4}}$$

2. $y = x \arctan(2x^3)$

$$y' = (1) \arctan(2x^3) + (x) \cdot \frac{6x^2}{1+(2x^3)^2}$$

$$y' = \arctan(2x^3) + \frac{6x^3}{1+4x^6}$$

3. $y = (\arcsin x)^2$

$$y' = 2(\arcsin x) \cdot \frac{1}{\sqrt{1-x^2}}$$

$$y' = \frac{2 \arcsin x}{\sqrt{1-x^2}}$$

4. $g(x) = \arcsin\left(\frac{1}{x}\right)$

$$g(x) = \arcsin(x^{-1})$$

$$g'(x) = \frac{-x^{-2}}{\sqrt{1-(x^{-1})^2}}$$

$$g'(x) = \frac{-1}{x^2 \sqrt{1-\frac{1}{x^2}}}$$

5. $f(x) = \arctan(e^{2x})$

$$f'(x) = \frac{e^{2x} \cdot 2}{1+(e^{2x})^2}$$

$$f'(x) = \frac{2e^{2x}}{1+e^{4x}}$$

6. $f(x) = \arcsin(4^x)$

$$f'(x) = \frac{4^x \cdot \ln 4}{\sqrt{1-(4^x)^2}}$$

$$f'(x) = \frac{4^x \cdot \ln 4}{\sqrt{1-4^{2x}}}$$